Improved Variable Step Size NLMS Algorithm

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Abstract: Aiming at the shortcomings of the traditional normalized least mean square (NLMS) algorithm, an improved variable step size NLMS algorithm is proposed to improve the problem caused by the use of fixed step size, that is contradiction between convergence speed and steady-state error. The proposed algorithm in this paper establishes the relationship between the step size and the cross-correlation function of the error signal and the input signal, which can significantly improve the convergence speed and has a small steady state error. At the same time, when the filter system is abrupt, the algorithm can fast convergence and has good tracking performance.

1. Nlms Algorithm And Analysis

1.1. Algorithm Principle

The normalized least mean square (NLMS) adaptive filtering algorithm can be regarded as a performance improvement of the LMS algorithm. It can be regarded as a time-varying step size LMS algorithm and replaces the LMS algorithm in practical applications. The NLMS algorithm uses a normalized method to avoid amplification interference of gradient noise and has better convergence when the input signal is large.

Taking an adaptive filter as an example, x(n) is the input signal, e(n) is the error signal, y(n) is the output signal, and d(n) is the desired signal. The core formula is:

$$\mathrm{e(n)} = \mathrm{d(n)} - \mathrm{y(n)} \quad (1)$$

$$\mathrm{y(n)} = w^T(n)x(\mathrm{n}) \quad (2)$$

$$\mathrm{w(n+1)} = \mathrm{w(n)} + \frac{\mu(n)}{\varepsilon+\|X(n)\|^2}e(n)X(n) \quad (3)$$

Equation 3 is the formula for weight update, w(n) is the filter weight update. $\mu(n)$ is a fixed step size of the NLMS algorithm and the value range is 0 ~ 2.

1.2. Improved algorithm

The literature [1] proposed the use of atan(g) function to establish a functional relationship between the step size and the gradient, wherein the defined gradient is obtained by using the instantaneous signal estimation, and the estimation of the instantaneous signal is regarded as a cross-correlation function of the error signal e(n) and the reference signal x(n), the normalization algorithm is added to obtain the new NLMS algorithm with variable step size. The step size formula is as follows:

$$\mu(\mathrm{n}) = \mu_0\alpha atan(\beta\|e(n)x(n)\|) \quad (4)$$

Based on this method, under the criterion of mean square error, when the optimal solution in the filtering process of the adaptive filter algorithm is obtained, the cost function J of the mean square error is defined. The cost function J is at the minimum, it is the optimal solution. For this condition, there is an equivalent condition that is the principle of orthogonality:

$$E[x(n)e_0(n)] = 0 \quad (5)$$

Among them, $e_0(n)$ is the specific value of the estimation error under the optimal condition. Therefore, the cross-correlation function of e(n) and x(n) will decrease with the iteration, and will be large in the initial convergence stage of the algorithm, and when the algorithm tends to be stable, the value will be small, so it can be used to reflect changes in the step size.

Based on the above description, this paper proposes a new variable step size NLMS algorithm for the fixed step size factor in traditional NLMS, the weight update formula is as follows:

$$w(n+1) = w(n) + \frac{\mu(n)}{\varepsilon + \|x(n)\|^2} e(n)x(n) \quad (6)$$

Initially, $\alpha = 0.9$, $\beta = 0.8$, factor used to prevent signal mutations, $f(n) = 0$. And the variable step size formula is as follows:

$$\mu(n) = \frac{\alpha \|e(n)x(n)\|}{f(n)} \quad (7)$$

In Equation 3, f(n) is also updated with the error signal and the input signal, and the formula f(n) is defined as follows:

$$f(n) = \beta F(n) + \|e(n)x(n)\| \quad (8)$$

Equations 6, 7 and 8 are the weight update formulas of the variable step size NLMS algorithm proposed in this paper, which uses the Euclidean norm to simplify the calculation.

2. Performance Simulation Experiment

Through experimental comparison, it is proved that the improved NLMS algorithm in this paper has the best convergence effect under the minimum mean square error criterion. The test data uses a random analog speech signal with an adaptive filter order of 9 and a sampling point of 500.

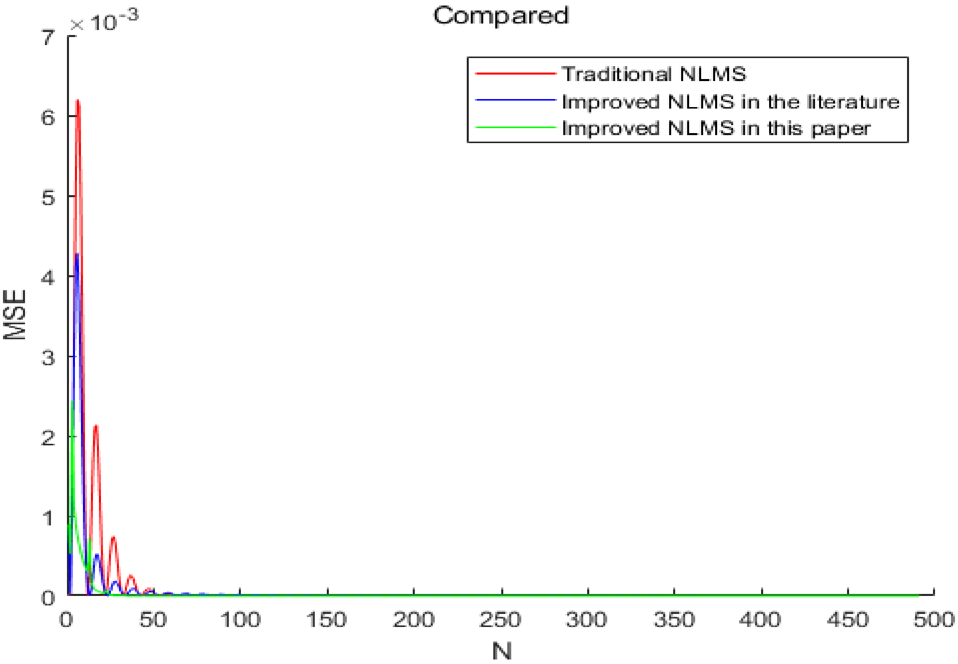

Figure 1. Algorithm Comparison.

As shown in Figure 1, the red line is the convergence curve of a fixed step factor $\mu = 0.3$; the blue line is the convergence curve of the improved NLMS algorithm in the literature [1]; the green line is the convergence curve of the improved NLMS algorithm in this paper, and $\alpha = 0.9$, $\beta = 0.8$, $f(n) = 0$. As can be seen from the figure, mean squared error of the improved algorithm were smaller than other contrast algorithms at the start, and then the converges level of

NLMS algorithm tends to be stable to about 50, and while improved NLMS algorithm tends to be stable to about 30.

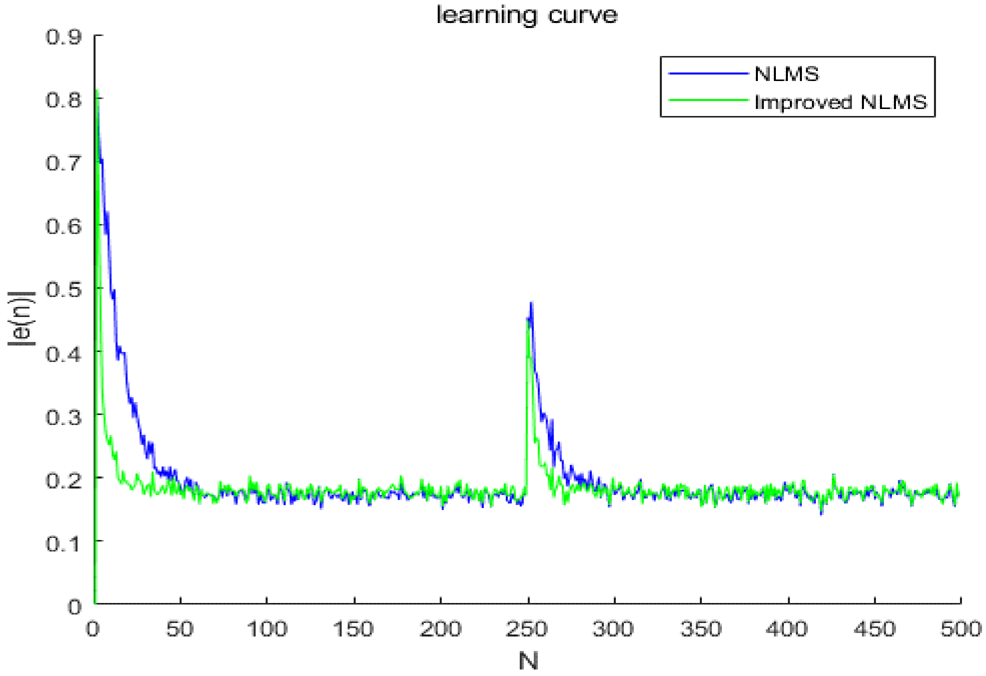

Figure 2. Algorithm Learning Curve.

In addition, the tracking ability of the verification algorithm, that is, after the signal is mutated, compares the NLMS algorithm with the improved NLMS algorithm. To achieve the same steady-state offset, adaptive filter order is 2, the sampling points is 500. Fixed step factor for NLMS algorithm $\mu = 0.1$,the improved NLMS algorithm $\alpha = 1, \beta = 0.85, f(n) = 0$.

As shown in Figure 2, the blue line is the traditional NLMS algorithm, and the green line is the improved algorithm in this paper. When n = 250, the system has a sudden change, as can be seen from Figure.2, the improved algorithm in this paper has faster convergence speed and stronger tracking ability than the traditional NLMS algorithm.

In summary, this paper proposed an improved NLMS algorithm, through the introduction of step length and the cross-correlation between the error signal and the input signal, it makes the algorithm achieve faster convergence when the smaller steady-state error is obtained. At the same time, it has better tracking ability, and it can better converge quickly in the face of sudden changes in system signals.

3. Conclusion

In the weight update of the traditional NLMS algorithm, the definition of variable step size is introduced. The step size is dynamically adjusted by the cross-correlation function of the error signal and the input signal. According to the simulation experiment, it can be seen that the improved NLMS algorithm in obtaining the small steady-state error, it's the convergence speed is also better than the traditional NLMS algorithm, and it has better tracking ability when the system introduces a sudden change.

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